

Please check that this question paper contains 9 questions and 2 printed pages within first ten minutes.

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Uni. Roll No.

Program: B.Tech. (Batch 2018 onward)

Semester: 2nd

Name of Subject: Mathematics-11

Subject Code: BSC-104

Paper ID:15940

MORNING

11 MAY 2023

Time Allowed: 03 Hours

Marks: 60

Max.

NOTE:

- 1) Parts A and B are compulsory
- 2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
- 3) Any missing data may be assumed appropriately

Part – A

[Marks: 02 each]

Q1.

- a) State Dirichlet's conditions for a function $f(x)$ to be expressed as Fourier series.
- b) Define concavity and convexity of a curve $y = f(x)$.
- c) Evaluate the first order partial derivatives of $u = \frac{x-y}{x+y}$.
- d) Evaluate $\iint (x^2 + y^2) dx dy$ over the region in the positive quadrant for which $x + y \leq 1$.
- e) If \vec{E} and \vec{F} are irrotational then prove that $\vec{E} \times \vec{F}$ is solenoidal.
- f) Evaluate $\int_0^2 \int_1^2 \int_0^{yz} xyz dx dy dz$.

Part – B

[Marks: 04 each]

Q2. If $u = \sin^{-1} \frac{x+y}{\sqrt{x+y}}$, Prove that: $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4 \cos^3 u}$.

Q3. Trace the curve $r = a(1 + \sin \theta)$ by discussing its features.

- Q4. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$.
- Q5. Discuss the Physical interpretation of curl of a vector point function.
- Q6. What is the directional derivative of the function $4xz^3 - 3x^2yz^2$ at the point $(2, -1, 2)$ along z- axis.
- Q7. Expand $f(x) = x^2$; $-\pi \leq x \leq \pi$ as a Fourier Series.

Part – C**[Marks: 12 each]**

- Q8. Verify Green's theorem for $\oint_c (xy + x^2)dx + (x^2 + y^2)dy$, where c is the boundary of the region defined by the lines $x = \pm 1, y = \pm 1$.

OR

- a) Find the smaller of the areas bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x + 3y = 6$, Using double integration. (6)
- b) Using triple integration find the volume of the tetrahedron bounded by $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (6)

- Q9. Expand $f(x) = x \sin x$; $0 \leq x \leq 2\pi$ as a Fourier Series.

OR

Use Lagrange's method to find the minimum value of $x^2 + y^2 + z^2$, given that $xyz = a^3$.
